TD 00-002

Reference Harmonics Table Version 3.0 for the Fermilab LHC IR Quadrupole (MQXB)

Jim Strait
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This note presents a new version 3.0 reference harmonics table for the MQXB, based on measurements from the most recent four model magnets HGQ05-08. The coils in this series are molded as close to the design size as seems reasonable to expect, and the magnets include only very small deviations in pole and mid-plane shim thicknesses from the design. The harmonics data are then taken, without correction, to be representative of those in the production series. Table I shows the measured body-field harmonics for the four magnets in this series[1]. These data were taken 6000 A (DC), where persistent current effects are minimal, and therefore represent essentially pure "geometric" harmonics. Table II shows the mean and rms deviation from the mean of the harmonics over the four magnets, and compares them with the values in version 2.0 of the reference harmonics table. The error bars quoted on the mean are the rms/ $N_{magnets}$ and on the rms are rms/ $(2N_{magnets})$, representing estimates of the standard deviations of these quantities in the Gaussian statistics limit. The rms deviation about zero is also included, since most the mean for most harmonics is statistically consistent with zero. The values in Table II are plotted in Figures 1 and 2, except that in Figure 2 the error bars on the rms values are ±45% confidence bounds according to an analysis using the Student's-t distribution[2].

Figure 1 shows that all the mean values, except for b_3, a_4 and a_6, are within the uncertainty range of Table 2.0. Two of these, b_3, and a_6 are a bit more than 3 sigma from zero, suggesting that these harmonics may have systematically non-zero values. The skew octupole a_4 is only 1.7 sigma from zero, and therefore most likely represents a statistical fluctuation of a distribution with a true mean of zero. Two other harmonics, b_5, and b_6, are a little over 2 sigma from zero, but with means within the uncertainty range. Modest efforts have been made to determine if the values of b_3, a_4 and a_6 can be correlated with systematic coil size deviations or consequences of specific construction procedures, but no conclusive results have been found. Thus it is uncertain whether any of the harmonics with non-zero means over this small distribution indicate that true systematic harmonics have been built into the model magnet series or not. If even b_3, and a_6 are statistical fluctuations, then a more accurate estimation of the underlying rms variation of the distribution is the rms about zero, rather than the rms about the mean. Both estimators are shown in Table II and Figure 2.

With the exception of b_3, a_4 and a_6, all harmonics are well within the uncertainty range, and the high order harmonics, from b_8 and a_8 up are all much smaller than the limits. Figure 3 is a histogram of the ratio of mean to uncertainty limits for low and high order harmonics. If we treat b_3, a_4 and a_6 as special cases, then it is clear that for the other harmonics, the uncertainty range on the mean is overestimated in Reference Table V2.0. A more realistic, but still conservative estimate would be about 75% of the V2.0 values for the low order harmonics and 40% or less for the high order harmonics.

Table I. Measured harmonics for HGQ05-08.

	HGQ05	HGQ06	HGQ07	HGQ08
b3	0.72	0.25	0.18	0.61
a3	0.12	-0.27	0.41	-0.01
b4	0.00	0.09	0.01	-0.12
a4	0.19	-0.31	-0.50	-0.44
b5	-0.04	-0.11	-0.04	-0.01
a5	0.05	-0.07	-0.24	0.12
b6	-0.30	-0.05	-0.45	-0.06
a6	-0.030	-0.050	-0.101	-0.031
b7	0.010	-0.030	0.017	-0.008
a7	0.010	0.000	0.075	-0.003
b8	0.000	0.000	0.001	-0.003
a8	0.000	0.000	0.011	-0.013
b9	0.000	0.000	-0.009	0.001
a9	0.000	0.000	0.010	0.009
b10	0.010	0.000	-0.016	-0.006
a10	0.000	0.000	-0.002	-0.004

Table II. Mean and rms variation of harmonics for HGQ05-08 compared with Reference Harmonics Table V2.0.

	Ref Table 2.0		HGQ05-08 Measured		RMS
	Uncert	Random	Mean	RMS	rel 0
b3	0.30	0.80	0.44 ±0.13	0.26 ±0.09	0.49
a3	0.30	0.80	0.06 ±0.14	0.28 ±0.10	0.25
b4	0.20	0.80	-0.01 ±0.04	0.09 ±0.03	0.08
a4	0.20	0.80	-0.26 ±0.16	0.31 ±0.11	0.38
b5	0.20	0.30	-0.05 ±0.02	0.04 ±0.02	0.06
a5	0.20	0.30	-0.03 ±0.08	0.16 ±0.06	0.14
b6	0.60	0.60	-0.22 ±0.10	0.19 ±0.07	0.27
a6	0.05	0.10	-0.05 ±0.02	0.03 ±0.01	0.06
b7	0.05	0.06	-0.003 ±0.010	0.021 ±0.007	0.018
a7	0.04	0.06	0.021 ±0.018	0.037 ±0.013	0.038
b8	0.03	0.05	-0.001 ±0.001	0.002 ±0.001	0.002
a8	0.03	0.04	-0.001 ±0.004	0.010 ±0.003	0.009
b9	0.02	0.03	-0.002 ±0.002	0.005 ±0.001	0.005
a9	0.02	0.02	0.005 ±0.002	0.006 ±0.002	0.007
b10	0.02	0.03	-0.003 ±0.005	0.011 ±0.003	0.010
a10	0.02	0.03	-0.002 ±0.001	0.002 ±0.001	0.002

The specific values chosen for Reference Table V3.0 are shown by the small hash marks in Figure 1. For most of the low order harmonics, 75% of the V2.0 values are taken, and a common value of 0.008 is taken for the high order harmonics. For b_3 and a_6, symmetric values equal to the sum of the mean plus one sigma on the mean are taken, and the value for a_4 is left unchanged.

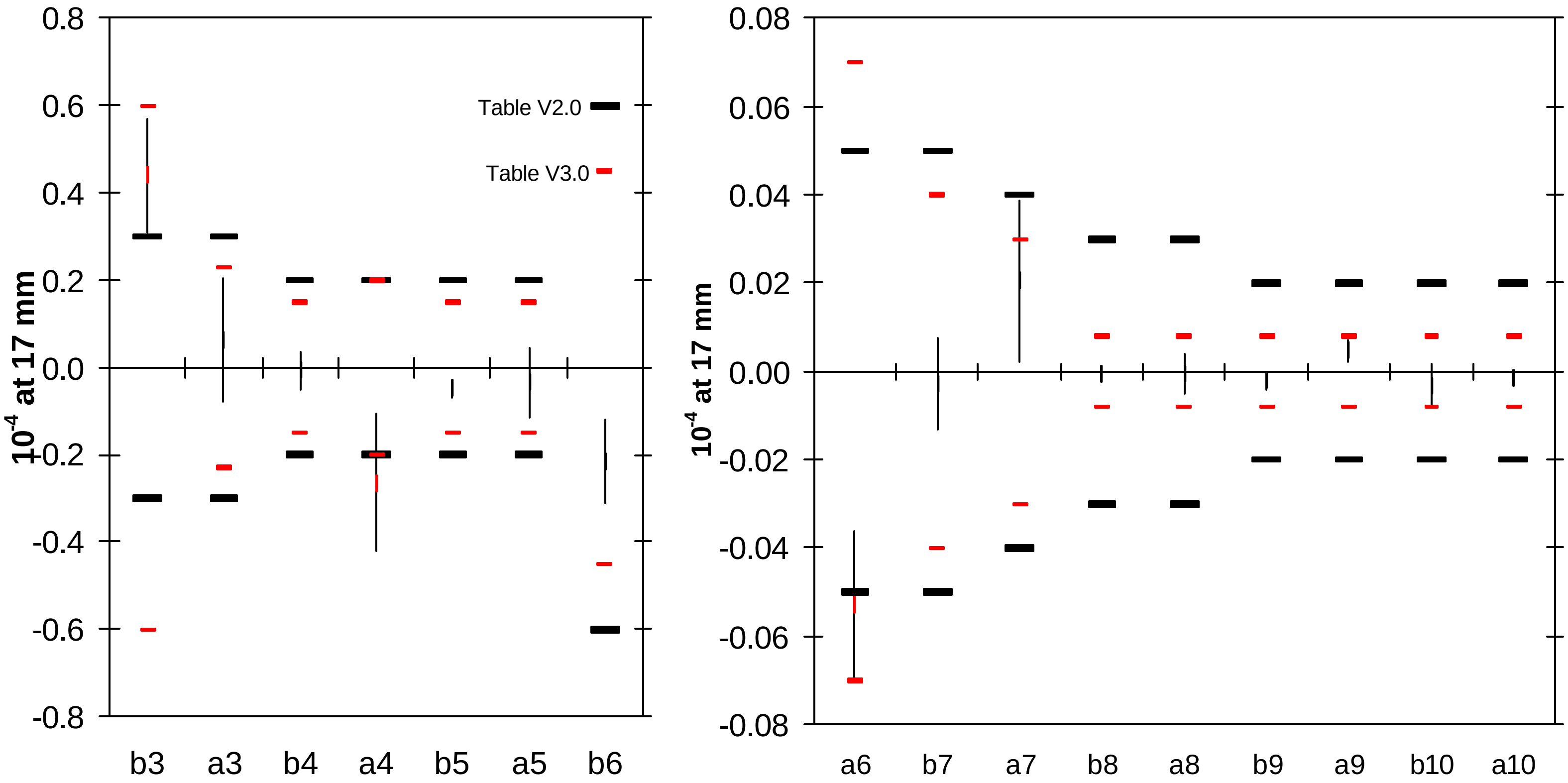

Figure 1. Mean value of harmonics for HGQ05-08. The error bars are the rms/ $\sqrt{N_{magnets}}$, the heavy horizontal lines show the uncertainty range in Reference Table V2.0, and the lighter (red) lines show the uncertainty range for V3.0.

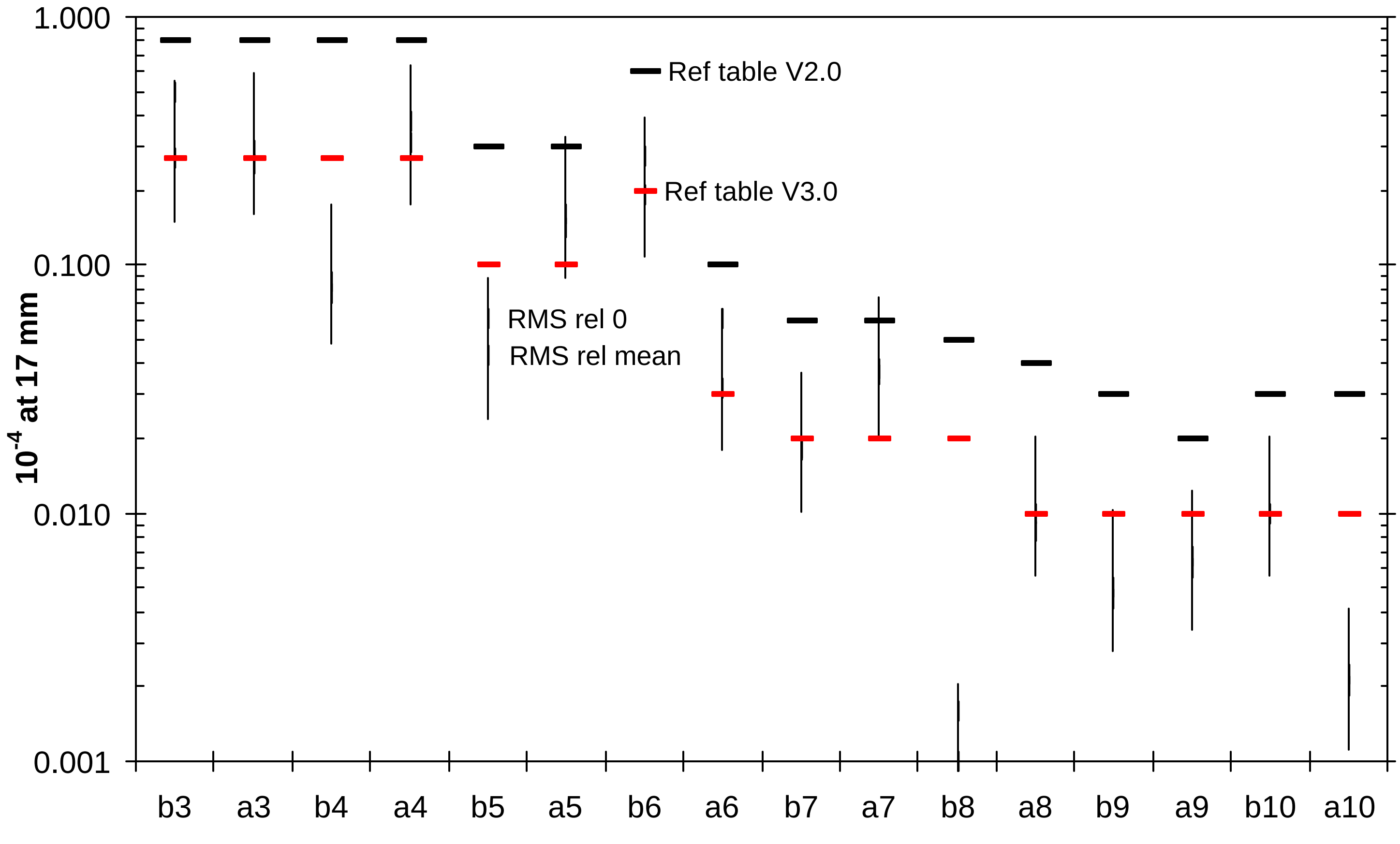

Figure 2. RMS variation of harmonics for HGQ05-08. The filled (open) symbols are the rms about the mean (about zero). The error bars show ±45% confidence bounds, the heavy (black) horizontal lines show the rms values in Reference Table V2.0, and the lighter (red) show the values for V3.0.

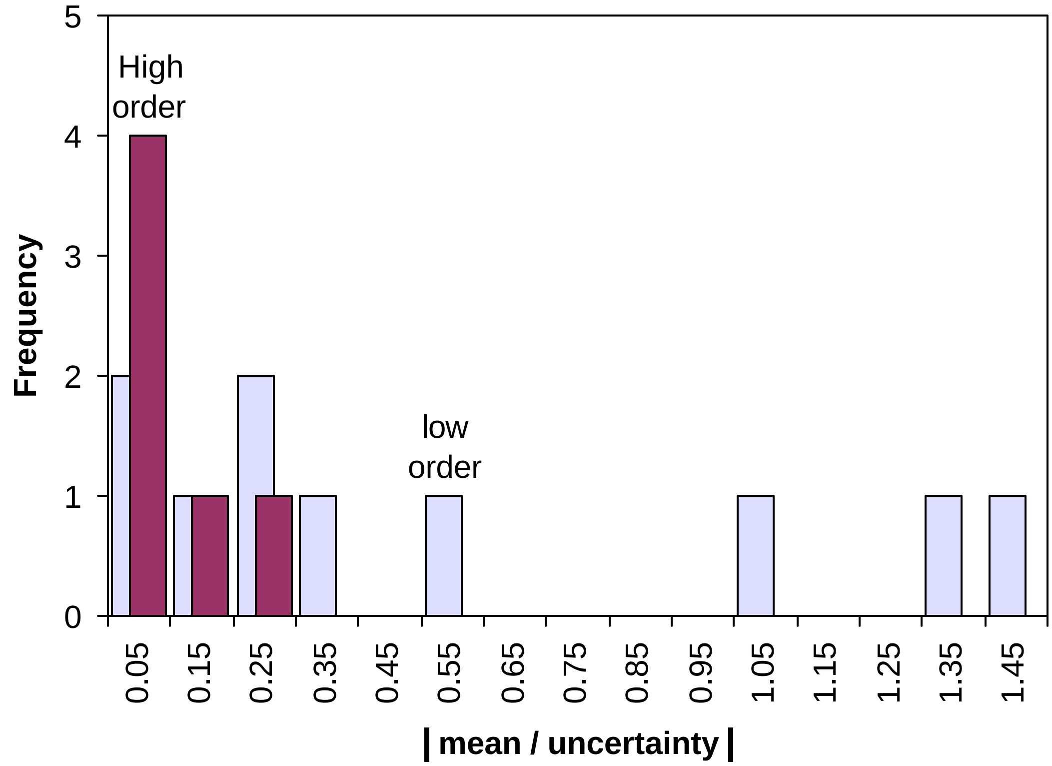

Figure 3. Histogram of the ratio of the absolute value of the mean to the absolute value of the uncertainty in Reference Table V2.0 for the lower order (b_3/a_3 - b_7/a_7) and higher order (b_8/a_8 – b_{10}/a_{10}) harmonics

Figure 2 shows the rms variation of the harmonics over the four magnets, computed both around the mean and around zero, together with a symmetric 90% confidence interval on the rms about the mean. The two rms values differ substantially only for the harmonics that have significantly non-zero means, principally b_3 and a_6. All rms values, computed either way, are smaller than the random error assumed in Reference Table V2.0, and almost all are substantially smaller. Within errors, the general pattern of the measured rms variations is the same as the values in the Reference Table only shifted down. Given the significant errors on the rms of such a small number of samples as an estimator of the true random variations of the underlying distribution, I believe that it is more robust to use the shape of the distribution of random errors as a function of harmonic number from Table V2.0, adjusted down to fit the data, than to use the raw values themselves as the basis for the revised error table. This procedure appears to give inconsistent results for only two harmonics, b_8 and a_{10}, whose measured rms is a much smaller fraction of the random error in Table 2.0 than the other harmonics. For the remaining harmonics, a table made by taking 34% of the values in Table V2.0 results in a good fit to the data, as shown by the lower set of hash marks in Figure 2. Note that over the 14 harmonics considered in such a “fit,” the adjusted random error values of 12, or 86%, lie within the 90% confidence bands. For the two “special cases,” b_8 and a_{10}, the same 34% factor is applied in the absence of understanding of why their rms variation is so much smaller over these 4 magnets..

The resultant Reference Harmonics Table V3.0 for the body field at high energy is shown in Table III. It represents a realistic estimate of the expected distribution of errors in the production series. In considering the required degree of conservatism applied in generating this table, we need to recall that, at this point, the major use of this table will be to specify the strengths of the multipole correction windings. The procedure used to compute the required strength[3] effectively allows for 4-5 sigma deviations on the operating strength found in the ensemble of tracking runs with different seeds, and therefore builds in additional conservatism. Thus the reference table need not, and should not, be overly conservative, so as to guard against over-specifying the corrector strengths.

We adopt this table as V3.0 now, not waiting for the analysis of data from HGQ09. If, when HGQ09 data are understood, and, as I hope, when it is understood whether the apparently non-zero values of b3 and a6, and possibly a4, represent true systematic errors or not, then this table can be revised to V3.1. It is my judgement that the additional data from one more magnet is unlikely to make substantial changes in the table, and additional understanding of the sources of the apparent non-zero harmonics would, at most, cause us to trade slightly smaller values of the uncertainty range against slightly larger values of the random errors. Thus the additional data are unlikely to change the specifications on corrector strengths.

Table III. Reference Harmonics Table V3.0 for the MQXB body at collision energy, compared with Table V2.0 and HGQ05-08 measured harmonics

	Ref Table 3.0		Ref Table 2.0		HGQ05-08 Measured	
	Uncert	Random	Uncert	Random	Mean	RMS
b3	0.60	0.27	0.30	0.80	0.44 ±0.13	0.26 ±0.09
a3	0.23	0.27	0.30	0.80	0.06 ±0.14	0.28 ±0.10
b4	0.15	0.27	0.20	0.80	-0.01 ±0.04	0.09 ±0.03
a4	0.20	0.27	0.20	0.80	-0.26 ±0.16	0.31 ±0.11
b5	0.15	0.10	0.20	0.30	-0.05 ±0.02	0.04 ±0.02
a5	0.15	0.10	0.20	0.30	-0.03 ±0.08	0.16 ±0.06
b6	0.45	0.20	0.60	0.60	-0.22 ±0.10	0.19 ±0.07
a6	0.07	0.03	0.05	0.10	-0.05 ±0.02	0.03 ±0.01
b7	0.04	0.02	0.05	0.06	-0.003 ±0.010	0.021 ±0.007
a7	0.03	0.02	0.04	0.06	0.021 ±0.018	0.037 ±0.013
b8	0.008	0.020	0.03	0.05	-0.001 ±0.001	0.002 ±0.001
a8	0.008	0.010	0.03	0.04	-0.001 ±0.004	0.010 ±0.003
b9	0.008	0.010	0.02	0.03	-0.002 ±0.002	0.005 ±0.001
a9	0.008	0.010	0.02	0.02	0.005 ±0.002	0.006 ±0.002
b10	0.008	0.010	0.02	0.03	-0.003 ±0.005	0.011 ±0.003
a10	0.008	0.010	0.02	0.03	-0.002 ±0.001	0.002 ±0.001

The discussion above has focused on the body-field harmonics at high field, since the body dominates the field quality considerations and the field quality in these magnets is most important under collision conditions. A complete reference table must also include injection errors for the body and field errors in the ends. Analysis of the end data is under way, and when complete new values will be entered into the next version of the error table. For V3.0 we use the same values as in V2.0.

At low field, in the most recent three model magnets, the field errors are dominated by eddy-current induced harmonics, resulting from low cross-over resistance in the cable generated by high cure pressure and temperature[4]. For the production series, a modified cure cycle that avoids simultaneous high temperature and pressure is expected to greatly reduce this effect. This will be tested in HGQ09. Thus, for the moment, we do not have sufficient data to include dynamic field errors in the injection harmonics table. The V3.0 injection table is generated from the collision table by including the non-zero value of b_6 and an additional uncertainty due to persistent currents. Further analysis of the static field errors at injection energy, together with the results of the tests of HGQ09, will be used to update the injection table in the future. The complete Reference Harmonics Table V3.0 is presented in Table IV.

Table IV. Reference Harmonics Table V3.0 for the MQXB. For injection conditions, only geometric and persistent current effects are included.

Magnet Body Harmonics (units, r_{ref} = 17 mm)

	Collision Energy			Injection Energy		
	Mean	Uncert	Random	Mean	Uncert	Random
b3	0	0.60	0.27	0	0.60	0.27
a3	0	0.23	0.27	0	0.23	0.27
b4	0	0.15	0.27	0	0.15	0.27
a4	0	0.20	0.27	0	0.20	0.27
b5	0	0.15	0.10	0	0.15	0.10
a5	0	0.15	0.10	0	0.15	0.10
b6	0	0.45	0.20	-0.84	0.60	0.20
a6	0	0.07	0.03	0	0.07	0.03
b7	0	0.04	0.02	0	0.04	0.02
a7	0	0.03	0.02	0	0.03	0.02
b8	0	0.008	0.020	0	0.008	0.020
a8	0	0.008	0.010	0	0.008	0.010
b9	0	0.008	0.010	0	0.008	0.010
a9	0	0.008	0.010	0	0.008	0.010
b10	0	0.008	0.010	0	0.008	0.010
a10	0	0.008	0.010	0	0.008	0.010

Magnet End Integrated Harmonics, Injection or Collision Energy (unit-m, r_{ref} = 17 mm)

	Lead End			Return End		
	Mean	Uncert	Random	Mean	Uncert	Random
A2	16.4					
B6	0.82	0.82	0.31	0	0.41	0.31
A6	0	0.21	0.06			
B10	-0.08	0.08	0.04	-0.08	0.08	0.04
A10	0	0.04	0.04			

References
[1] J. Kerby, private communication.
[2] J. Dimarco, private communication.
[3] J. Wei, *Workshop Summary*, W. Fischer, et al., *LHC Interaction Region Correction Scheme Studies*, Proceedings of Workshop on LHC IR Correction and Compensation, BNL, May 1999.
[4] N. Andreev, et al., *Field Quality in Fermilab-built Models of Quadrupole Magnets for the LHC Interaction Regions*, presented at MT-16, Jacksonville, FL, September 1999.