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Pion Decay: Theory

Outline

1.) $BR(\pi^+ \to \pi^0 e^+ \nu_e)$ and CKM Unitarity

2.) $R_{e/\mu} \equiv \dfrac{\Gamma(\pi \to e\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$ & e-μ Universality

3.) Random Comments

i) $\pi^+ \to e\nu\gamma$

ii) $\pi^0 \to \gamma\gamma$

iii) Goldberger-Treiman Relation $\sqrt{2} f_\pi g_{\pi pn} = (m_p + m_n) g_A$

Based on: Marciano & Sirlin PRL 71, 3629 (1993)

Czarnecki, Marciano & Sirlin 2004 PRD

1) $BR(\pi^+ \to \pi^0 e^+ \nu(\gamma))$ and CKM Unitarity

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ Unitarity Test

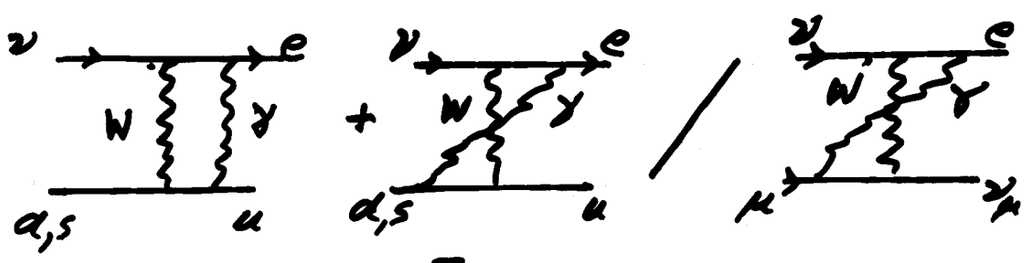
Normalize using $\tau_\mu \to G_\mu = 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$

Include EW Radiative Corrections

Large ~ 4%!

$|V_{ub}|^2 = 2.1(3) \times 10^{-5}$ Negligible

$|V_{ud}|^2 + |V_{us}|^2 \simeq 1$ Cabibbo Universality



$$1 + \frac{2\alpha}{\pi} \ln \frac{m_Z}{E_{max}} + \ldots \simeq 1.04$$

Superallowed $0^+ \to 0^+$ β-decay → $|V_{ud}| = 0.9740(5)$ Mainly Theory
(Range 0.9735 - 0.9745)
(Towner + Hardy)

PDG(2002) = 0.2196(26) K_{e3} ($K \to \pi e \nu$) Global Fit!

$|V_{ud}|^2 + |V_{us}|^2 = 0.9969(15)$ 2sigma deviation (20yrs)

V_{ud} or V_{us} Problem?

Other $|V_{ud}|$ determinations

Neutron Decay $n \to p e^- \bar{\nu}_e$ V+A

$$|V_{ud}| = \frac{4908(4)\,\text{sec}}{\tau_n (1+3g_A^2)}$$ Includes R.C. { New, Czarnecki, Marciano, Sirlin

$\tau_n^{ave} = 885.7(7)\,\text{sec}$
$g_A^{ave} = 1.2720(18)$ $\rightarrow$ $|V_{ud}| = 0.9729(4)(11)(4)_{RC}$ Bright Future

Pion Beta Decay $\pi^+ \to \pi^0 e^+ \nu_e$ $BR \sim 10^{-8}$! Hard } Clear

PSI exp $\rightarrow$ $|V_{ud}| = 0.9749(26)\left[\frac{BR(\pi^+ \to e^+\nu(\gamma))}{1.2352 \times 10^{-4}}\right]$

PDG $BR(\pi^+ \to e^+\nu(\gamma)) = 1.230(4) \rightarrow |V_{ud}| = 0.9728(30)$

Small Theory Unc. ± 0.0002 Approx } Do Better?

$|V_{ud}| = 0.9740(5)$ Average $0^+ \to 0^+$ dominate

The K_{e3} revolution BNL E865 $K^+ \to \pi^0 e^+ \nu(\gamma)$
KTeV at FNAL $K_L \to \pi^+ e^- \bar{\nu}(\gamma)$
Increase by ~5%!

$|V_{us}| = 0.2259(12)(20)$ New K_{e3}

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9997(10)(10)$

Perfect Unitarity! The End?
Quit while your ahead?

Czarnecki, Marciano & Sirlin

A comparison of the different determinations of $|V_{us}|$ is illustrated in Fig. 1.

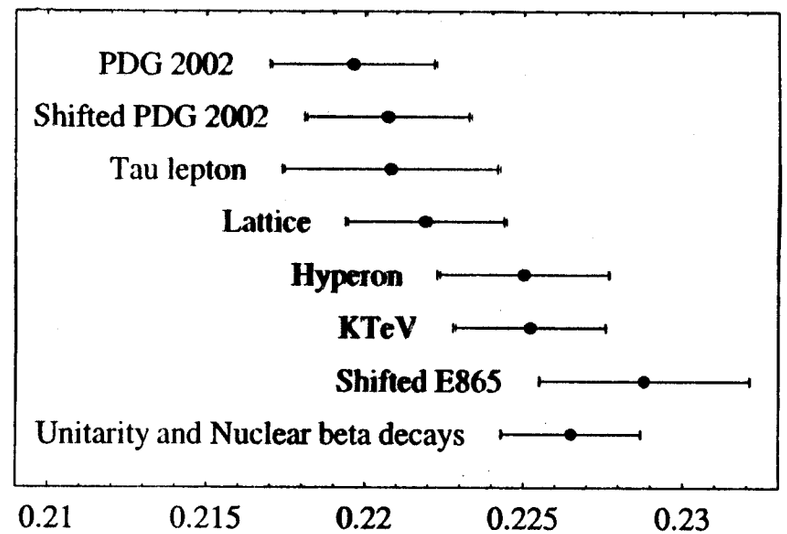


FIG. 1: Determinations of $|V_{us}|$ from various sources. The Hyperon value does not include theory errors. Shifted values correspond mainly to the change of linear to quadratic form factor parametrization. All K_{l3} results assume Eqs. (33,34).

<u>Still issues</u>: Some lower $|V_{us}|$ values (see Fig)

τ_n + g_A issues (Big discrepencies!)

Very High Stat. $\pi^+ \to \pi^0 e^+ \nu$ Still Warranted

But Hard

<u>Lattice</u> $f_K/f_\pi = 1.210(4)_{stat}(13)_{syst.} \to \underline{|V_{us}| = 0.2219(25)}$

$\underline{\Gamma(K^+ \to \mu\nu(\gamma))}/\Gamma(\pi \to \mu\nu(\gamma))$ τ_K + $BR(K \to \mu\nu)$ (Changes?)

<u>Will eventually dominate ± 0.0006!</u>

Nail Unitarity → Constrain New Physics

eg $BR(\mu \to e$ + wrong neutrinos)

Babu + Pakvasa LSND

2.) $\underline{R_{e/\mu} \equiv \dfrac{\Gamma(\pi^+ \to e^+ \nu(\gamma))}{\Gamma(\pi^+ \to \mu^+ \nu(\gamma))}}$ + e-μ Universality

TRIUMF $R_{e/\mu} = 1.2265(34)(44) \times 10^{-4}$

PSI $R_{e/\mu} = 1.2346(35)(36) \times 10^{-4}$

PDG Ave $= 1.230(4) \times 10^{-4}$

Theory (Including R.C.) Kinoshita; Marciano + Sirlin

$\underline{R_{e/\mu} = 1.2352(5) \times 10^{-4}}$ Very Clean! (Theorem)

$$\sim \frac{m_e^2}{m_\mu^2} \to \frac{m_e^2(m_\pi)}{m_\mu^2(m_\pi)} \quad \text{Running Masses}$$

$1.28347 \times 10^{-4} \to 1.2352(5) \times 10^{-4}$

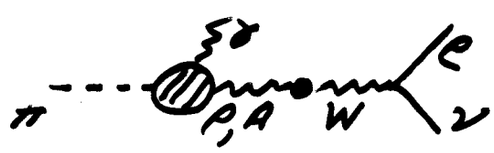
Deserves New Exp → $\pm 0.0005 \times 10^{-4}$ { keep up with Theory

$BR(\pi^+ \to e^+\nu(\gamma))$ also important for $\pi^+ \to \pi^0 e^+ \nu(\gamma)$
Normalization

Both $\pi^+ \to \pi^0 e^+ \nu(\gamma)$ + $\pi^+ \to e^+ \nu(\gamma)$
Worth High Precision
But Hard

3.) <u>Random Comments</u>

i) <u>$\pi^+ \to e^+ \nu \gamma$</u> Structure Dependent



+ ordinary Brem.

Tensor Int.?
Why?

ii) <u>$\Gamma(\pi^0 \to \gamma\gamma)$</u>

$f_{\pi^+} = 130.7 \pm 0.1$ MeV from $\pi^+ \to \mu^+ \nu(\gamma)$

$\to \Gamma(\pi^0 \to \gamma\gamma) \equiv 7.73(1)$ eV (PCAC) Anomaly

PDG Ave $= 7.74(55)$ eV Perfect!

<u>Best Exp $\to$ 7.25(23) eV</u> 2sigma (1985)

Anything Better?
More Recent?

iii) <u>Goldberger - Treiman Relation</u>

$$\Delta_\pi = 1 - \frac{(m_n + m_p)}{\sqrt{2} f_\pi g_{\pi pn}} g_A$$

$g_A = 1.2703(8)$ from $|V_{ud}|$ & τ_n

$f_\pi = 130.7(1)$ MeV

<u>$g_{\pi pn} = 13.04(6)$ or $13.4(1)$</u> Ambiguous?

$\Delta_\pi = 0.01$ or 0.036 unc $\sim \pm 0.005$

Expect $\Delta_\pi \sim 1\% \sim \frac{m_u + m_d}{2m_p}$

<u>What is g_{npn}</u>?

<u>The End</u>